

2017 RatioReport

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Deputy Director

The Honorable Larry Hogan
and
The General Assembly of Maryland
As required by Section 2-202 of the Tax-Property Article of the Annotated Code of Maryland, I am pleased to submit the Department of Assessments and Taxation's 2017 Assessment Ratio Report. This report measures the quality of real property assessments in each of Maryland's 24 jurisdictions.

The Department has adopted the national standards for measuring property assessment quality as outlined by the International Association of Assessing Officers. Those national standards, as well as our compliance with those standards, are discussed in the body of this report. Statewide, the Department has met the IAAO standard for coefficient of dispersion, indicating an overall uniformity of assessments.

Our entire team is committed to provide the customers we serve with the highest level of courteous, prompt and efficient service. I hope the information contained in this Report is of value to you and your constituents. As always, we welcome and appreciate your feedback and comments as to how the Department can enhance the level of service provided to our customers.

Sincerely,


Michael Higgs
Director

## 2017 ASSESSMENT RATIO REPORT

## SECTION I - OVERVIEW

The Department of Assessments and Taxation appraises real property for the purposes of property taxation. Properties are valued using the three approaches generally recognized by the appraisal profession: cost, sales comparison, and (when applicable) income.

Residential property characteristics include size, type and condition of structure, type and quality of construction, and any new improvements. Commercial properties are reviewed for size, type and condition of structure, type and quality of construction, any new improvements, current use of the property, types of tenants, and vacancy.

This year, the Department valued more than 758,770 properties, which required the use of mass appraisal techniques. While a fee appraiser is concerned with valuing one property at a time, an assessor is valuing whole neighborhoods. To accomplish this, special mass appraisal procedures are used. The assessor will review the data and calculate replacement costs for improvements much like a fee appraiser. The assessor will then review the sales from the area. In Maryland, the county's local assessment office receives a copy of all deeds and property sales prices when the deed transferring the property is recorded with the clerk of the court. In Baltimore City, the Department of Public Works provides that data to the Department. In the assessor's review and analysis of the sales, the assessor will develop land rates, depreciation tables, and sales analysis reports. After completing the analysis, the assessor applies the factors uniformly throughout the neighborhood to value all comparable properties in a uniform manner. Rental rates, vacancy and collection loss, expense ratios and capitalization rates are analyzed, and uniformly applied for comparable income producing properties.

The Department's work is reviewed by legislative auditors and is often scrutinized by individual property owners. We are continually striving for higher quality in assessment uniformity. Our quality control program begins with the individual assessor and the assessor's immediate supervisor. As work is completed, each assessor's supervisor reviews the analysis, makes recommendations, and approves the work. When the assessor completes the revaluation, the supervisor makes a random check using procedural and data editing. Following the completion of the revaluation, various computer edits are made to assure good valuation quality.

A measurement of quality is the assessed value/sale price ratio, which measures how closely the Department's values compare to the actual sales prices. Although the average assessed value/sale price ratio indicates a typical level of value, the marketplace is not perfect and there will always be properties that sell for more or less than can be anticipated. This may be due to factors such as buyers willing to pay extra for a unique property or declining values in a buyer's market.

In mass appraisal and assessment ratio studies, we are not only concerned with average assessed value/sale price levels (ratios) but also with the degree of spread (variation) from the typical ratio. The measurement of variation is called the coefficient of dispersion (COD). The lower the COD, the more uniform the assessment level.

In the balance of this report, Section II will give a more detailed explanation of the statistical terms as applied to assessment administration and quality control. Section III explains the

International Association of Assessing Officers' Standard of Performance for ratio studies. Section IV gives an overview of statewide appraisal quality for the most recent valuation of triennial Group 2, performed for January 1, 2017.

## SECTION II - RATIO STATISTICS

The purpose of this ratio study is to test the quality of the assessment product, which is examined from both an assessment level and assessment uniformity standpoint. Assessment level examines the degree to which the assessments are performed based upon the statutory requirement of full market value. Assessment uniformity measures the degree to which different properties are assessed at equal percentages of their market values. From our most recent valuation, the Department performs many ratio studies examining neighborhoods, types of structures, age of structures, etc.

Several measures of central tendency are used as performance gauges and are affected differently by outliers. A ratio of assessed value to sale price is calculated for each property, with the average ratio being the total of all ratios divided by the number of sales. The average (mean) ratio has a natural upward bias, indicating a higher level of assessment than has actually occurred. The median is the midpoint of any data listed from lowest to highest, and the median ratio is the point where half the ratios fall above and half ratios fall below. The median ratio counts each ratio equally. It is less biased by extreme ratios (outliers) or by individual property values. The weighted ratio is the total of all assessed values divided by the total of all sale prices. Since the weighted ratio counts each dollar equally, it is swayed by higher priced properties.

In addition to the general level of assessments, The Department is also concerned with the relative spread or variation that individual ratios fall from the typical. This variability is measured in two ways: coefficient of dispersion and coefficient of variation. These statistics measure horizontal inequities, or the dispersion of ratios regardless of the value of the individual properties. The coefficient of dispersion is calculated by dividing the average absolute deviation by the median ratio. The average absolute deviation is calculated by subtracting the median ratio from each ratio, adding all the results while ignoring positive and negative signs, and dividing that result by the number of ratios. Acceptable coefficients of dispersion depend on property type but should typically be $20 \%$ or less. Coefficient of variation is calculated by dividing the standard deviation by the mean or average ratio and multiplying by 100 . The variance is calculated by subtracting the mean from each ratio, squaring the differences, summing the squared differences, dividing by the total number of ratios less one. The standard deviation is calculated by taking the square root of the variance. The coefficient of dispersion is the preferable measure of variance unless a sample is normally distributed. In a normal distribution situation, coefficient of variation is the preferable measure of variance.

Another statistical measure used to gauge assessment uniformity is the Price Related Differential (PRD). The PRD tests to see if higher or lower valued properties are assessed at the same level, and is calculated by dividing the average ratio by the weighted ratio. This statistic measures vertical inequities. When low-value properties are valued at a higher percentage of their market value, the property taxes levied against these assessments would be considered regressive. Conversely, if high-value properties are valued at a higher percentage of their market value, property taxes levied against these assessments would be considered progressive. Typically,

PRDs have an upward bias because higher priced properties are more unique. PRDs should range between 0.98 and 1.03 , except for very small samples. For example, a PRD of 1.03 indicates undervaluation of high priced properties, while a PRD of .98 shows an under valuation of low priced properties.

Other descriptive statistical methods that may be used to analyze the assessment product are histograms, frequency distributions, and scatter diagrams. Due to the scope of this report, we have not examined them here. For further information on statistics relating to assessments, please refer to the International Association of Assessing Officers' publication "Improving Real Property Assessment".

Table I is the Fiscal Year 2017 Real Property Base/Ratio by Subdivision with assessment ratios expressed relative to full value. Table II is a history of weighted assessment ratios converted to full value ( $100 \%$ levels) that allows for comparison between years by adjusting for statutory changes in the assessment level. Table III displays examples of the statistical calculations used in this report.

Tables IV and V show the residential and commercial 2017 Ratio Study data by jurisdiction at assessed full market value level for the area most recently assessed. Following the ratio study is Table VI of the report detailing issues of assessment and appraisal quality that are summarized in Section IV.

## SECTION III - RATIO STUDY STANDARDS VALUES TO SALE PRICES

The International Association of Assessing Officers (IAAO) is a professional organization of assessing officials which provides educational programs, assessment administration standards, and research on appraisal and tax policy issues. IAAO has developed numerous standards and texts on appraisal and assessment administration. Additionally, the organization is a founding member of the national Appraisal Foundation which developed the Uniform Standards of Professional Appraisal Practice (USPAP).

IAAO's Standard on Ratio Studies was first published in September 1980 and was revised in April 2013. The Standard is advisory in nature, and provides guidance to those performing ratio studies in the mass appraisal field regarding the design, statistics, performance measures, and other issues related to such studies. The Maryland Department of Assessments and Taxation uses the fundamental ratio statistical measures of the Standard and has adopted IAAO's Assessment Ratio Performance Standard as the criteria to judge the performance of Maryland revaluations.

The IAAO Ratio Performance Standards are:
Ratio Study Uniformity Standards Indicating Acceptable General Quality*

| General Property Class | Jurisdiction Size /Profile /Market Activity | Max COD |
| :--- | :--- | :--- |
| Residential improved <br> (single family dwellings, <br> condominiums, manuf. <br> housing, 2-4 family units) | Very large jurisdictions / densely populated / newer properties / active markets | 5.0 to 10.0 |
|  | Large to mid-sized jurisdictions / older \& newer properties / less active markets | 5.0 to 15.0 |
| Income-producing <br> properties (commercial, <br> industrial, apartments,) | Rural or small jurisdictions / older properties / depressed market areas | 5.0 to 20.0 |
|  | Very large jurisdictions / densely populated / newer properties / active markets | 5.0 to 15.0 |
|  | Large to mid-sized jurisdictions / older \& newer properties / less active markets | 5.0 to 20.0 |
| Residential vacant land | Rural or small jurisdictions / older properties / depressed market areas | 5.0 to 25.0 |
|  | Large to mid-sized jurisdictions / slower development / less active markets | 5.0 to 15.0 |
|  | Rural or small jurisdictions/ little development / depressed markets | 5.0 to 25.0 |
| Other (non-agricultural) <br> vacant land | Very large jurisdictions / rapid development / active markets | 5.0 to 20.0 |
|  | Large to mid-sized jurisdictions / slower development / less active markets | 5.0 to 25.0 |
|  | Rural or small jurisdictions/ little development / depressed markets | 5.0 to 30.0 |

These types of property are provided for general guidance only and may not represent jurisdictional requirements.
*The COD performance recommendations are based upon representative and adequate sample sizes, with outliers trimmed and a $95 \%$ level of confidence.
*Appraisal level recommendation for each type of property shown should be between 0.90 and 1.10.
*PRD's for each type of property should be between 0.98 and 1.03 to demonstrate vertical equity.
$P R D$ standards are not absolute and may be less meaningful when samples are small or when wide variation in prices exist. In such cases, statistical tests of vertical equity hypotheses should be substituted.
*CODs lower than 5.0 may indicate sales chasing or non-representative samples.
Source: Standard on Ratio Studies; International Association of Assessing Officers; Kansas City, MO; April 2013; pg 34.
Ratio studies may be performed for various reasons including appraisal accuracy and assessment equity studies, to judge the need for management of a reappraisal, to identify problems with appraisal procedures, to assist in market analysis, and to adjust appraised values. Many ratio study design issues must be considered depending on the purpose of the ratio study.

This study considers unadjusted sales price data six months prior to and six months after the date of finality (date of valuation, January $1^{\text {st }}$ ) for which assessments have become effective so that an unbiased estimate of assessment performance can be obtained. Sales that are arms-length transactions between willing and informed buyers and sellers are used in this study. Maryland's ratio performance is good and conforms to the IAAO Standard.

While several measures of central tendency are calculated (average, median, and weighted ratios), the median is less affected by extreme ratios. The IAAO observes in its Standard that the median is generally the preferred measure of central tendency for monitoring appraisal performance. For this reason, median ratios are used in this study to measure compliance with IAAO standards.

As a proxy for time adjustments, this report uses sales from six months before the date of finality to six months after the date of finality. Under normal circumstances, with steadily changing
property values, these sales will balance. In unusual circumstances, when property values are rapidly changing, this will affect the ratio statistics.

On average, the residential values in this group increased by $6.4 \%$ and commercial property values showed an increase in 19 of the 24 subdivisions, with an overall average increase of 13.6 \% statewide.

Property value changes varied by region in the state since the last triennial revaluation in January, 2014.

Statewide, the Department met the IAAO standard for coefficient of dispersion indicating an overall uniformity of assessments.

Commercial properties are generally less similar than residential properties. Many commercial properties are income producing and are valued using the income approach to value. Most commercial uses are cyclical in nature. Various segments of the commercial real estate market may be ascending in value as a class, while others may be declining in market popularity. Because of the uniqueness of commercial and industrial properties, measures of central tendency tend to vary more widely than with residential properties.

The number of commercial properties is small compared to the number of residential properties. In several jurisdictions, the number of commercial properties which have sold is so small that the statistical measures are prone to bias. Allegany, Calvert, Caroline, Carroll, Charles, Dorchester, Garrett, Kent, Queen Anne's, St. Mary's, Somerset and Talbot Counties all had fewer than 10 arms-length commercial transfers for Group 2. In those jurisdictions, individual statistical measures would be unreliable due to sample size.

The number of commercial sales increase from 441 statewide in the 2016 Ratio Report to 580 statewide in the 2017 Ratio Report.

## SECTION IV - STATEWIDE COMPARISON OF DEPARTMENT'S VALUES TO SALE PRICE

Quality is the degree of excellence of a product or service as determined by the extent to which they measure up to certain standards. In this case, a measure of quality is the ratio study measuring whether the assessor appraised properties uniformly at market value. The ratio study conducted in this report is based upon sales data occurring, for the most part, after the time period of sales used by the assessor in the group of properties being reassessed.

Assuming the assessor applied the mass appraisal model uniformly to all properties, this ratio study should show uniformity of assessment. This ratio study is a cross check by Department management to assure quality of the mass appraisal work product. The ratio statistics for each county in Table IV was conducted on 25,456 improved residential property sales from July 1, 2016 to June 30, 2017 and compares the Department's valuations to sale prices.

The frequency distribution in Table IV and statistics following present a statewide ratio analysis of improved residential property sales from July 1, 2016 to June 30, 2017 comparing the Department's values to sales prices. The measures of central tendency indicate that properties are
valued at approximately $93.5 \%$ of sale price and that on average all other properties have very similar ratios as indicated by the 8.55 Coefficient of Dispersion. Additionally, higher valued properties are assessed at a similar level to lower valued properties as indicated by a Price Related Differential statistic of 1.00 . A price related differential of 1.00 indicates vertical uniformity across all strata of property values.

The analysis from Table IV and the following descriptive statistics indicates that values determined by assessors for the most recent triennial Group 2 valuation attained a uniform and appropriate level of value. At the time of valuation, the assessments were close to the sale price.

In summary, the data shows that properties throughout the State are assessed uniformly as required by law.

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| $\left\lvert\, \begin{gathered} \infty \\ \stackrel{\infty}{\circ} \end{gathered}\right.$ | $\left\|\begin{array}{c} \stackrel{\infty}{i} \\ \stackrel{\sim}{2} \end{array}\right\|$ |  | $\left\lvert\, \begin{aligned} & \infty \\ & \substack{a \\ i} \end{aligned}\right.$ |  |  | $\begin{aligned} & \infty \\ & \\ & \hline \end{aligned}$ |  | $0$ |  | $$ | $\stackrel{\infty}{\infty} \underset{\infty}{\infty}$ | $\begin{array}{l\|l} \infty & \infty \\ \infty & \cdots \\ \infty & \vdots \end{array}$ | $\begin{array}{c\|c} \infty & \infty \\ n_{n} & \infty \\ 0 & \\ \hline \end{array}$ | $\left\|\begin{array}{l} \infty \\ \underset{y}{\bullet} \end{array}\right\|$ | $\stackrel{9}{9}$ | $\stackrel{\infty}{+}$ | $\stackrel{\infty}{\circ}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{0}{n} \stackrel{\infty}{\stackrel{\infty}{\sim}} \underset{\sim}{\infty}$ | $\underset{\sim}{\infty} \underset{\sim}{\infty} \underset{\sim}{\infty}$ | $\begin{array}{l\|l\|l} 0 \\ 0 & \infty \\ 0 \\ 0 \end{array}$ | H | $\left\|\begin{array}{c} \infty \\ \stackrel{\infty}{i} \end{array}\right\|$ | $\begin{aligned} & 8 \\ & \stackrel{\rightharpoonup}{6} \end{aligned}$ | $\left\|\begin{array}{c} n \\ 0 \\ 0 \end{array}\right\|$ |  |  |
| $\mid \underset{\substack{\infty \\ \underset{\sim}{2} \\ \hline}}{ }$ | $\|\stackrel{\infty}{\stackrel{\infty}{*}}\|$ | $\left\lvert\, \begin{gathered} \infty \\ \substack{\infty \\ \hdashline \\ \hline} \end{gathered}\right.$ | $\begin{aligned} & \circ \\ & \hline 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 8 \\ & \hline \end{aligned}$ | $\begin{array}{c\|c} \infty & \infty \\ \hdashline & \stackrel{\infty}{i} \end{array}$ | $\underset{\sim}{\infty}$ | $\begin{array}{c\|c\|c} \infty \\ \infty & \infty \\ \hline \end{array}$ | $\bigcirc$ | $\begin{gathered} \infty \\ - \\ - \\ \hline \end{gathered}$ | $\begin{array}{c\|c} 0 & \infty \\ 0 & \infty \\ 0 & 0 \\ 0 \end{array}$ | $$ | $$ | $\infty$ | $\left\lvert\, \begin{aligned} & \circ \\ & 0 \\ & \hline \end{aligned}\right.$ | $\left\|\begin{array}{l} 0 \\ 0 \\ i \end{array}\right\|$ | $0 \left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}\right.$ | $0$ | $\underset{\sim}{\infty}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{l\|ll} \infty \\ 0 & \infty \\ 0 & \ddots \end{array}$ |  | $0$ | $\begin{array}{lll} 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\left\|\begin{array}{c} 0 \\ \underset{i}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} n \\ \hat{0} \\ 0 \end{array}\right\|$ |  |  |
| $\|\stackrel{\circ}{\circ}\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $0$ | $\left\|\begin{array}{l} 0 \\ 0 \end{array}\right\|$ | $0$ |  |  | $0$ | $\div$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{l\|l} \infty & \stackrel{8}{8} \\ 0 & \stackrel{0}{0} \end{array}$ | $\begin{array}{l\|l} \hline 8 & 0 \\ \hdashline & 0 \\ \hline \end{array}$ | $\begin{array}{l\|l} 0 \\ \hline 0 & 0 \\ \hline \end{array}$ | $\begin{array}{ll} 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\stackrel{2}{\circ}$ | $\div$ | $\begin{array}{l\|l} 0 \\ 0 \\ 0 \end{array}$ | $0$ | $0$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $\left\lvert\, \begin{aligned} & \circ \\ & \stackrel{0}{0} \end{aligned}\right.$ | $\left\|\begin{array}{c} 0 \\ 0 \end{array}\right\|$ | $\circ$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | 苍 |  |  |
| $\left\|\begin{array}{l} 0 \\ \hdashline \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} \circ \\ i \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ i \end{array}\right\|$ |  | $$ |  | $\begin{array}{lll} 0 & 0 \\ 0 & \circ \\ 0 \end{array}$ | $8 \text { P }$ | $\begin{array}{lll} \infty & 0 \\ 0 & 0 \\ \hline-1 \end{array}$ | $\begin{array}{l\|l} \circ & 0 \\ i & i \\ i \end{array}$ |  | $\circ$ | $0$ | $\left\|\begin{array}{l} \infty \\ \underset{\sim}{\infty} \end{array}\right\|$ | $\left\|\begin{array}{l} \circ \\ 0 \end{array}\right\|$ | $8: \stackrel{\circ}{2}$ | $8$ | $0$ | $\begin{aligned} & 0 \\ & i \\ & i \end{aligned}$ | $\begin{array}{lll} 0 \\ 0 & \ddots \\ 0 & \ddots \end{array}$ | $0$ | $\stackrel{\circ}{\stackrel{\circ}{4}}$ | $\left\|\begin{array}{c} 0 \\ \underset{i}{0} \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \tilde{0} \\ & \underset{\infty}{2} \end{aligned}\right.$ |  |  |
| $\left\|\begin{array}{l} \circ \\ +0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \infty \\ & 0 \\ & 0 \end{aligned}\right.$ | $\underset{\infty}{\infty}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{ll} \substack{0 \\ \hline \\ \hline \\ \hline} \\ \hline \end{array}$ | $\begin{array}{l\|l} 8 & 0 \\ 0 & 0 \end{array}$ |  | $0 \mid \stackrel{\circ}{\circ}$ | $\left.\begin{array}{l} \circ \\ 0 \end{array}\right)$ |  | $\begin{array}{l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|l\|} \hline \end{array}$ | $\begin{gathered} 0 \\ \hline 0 \\ \hline \end{gathered}$ | $\begin{array}{l\|l} 0 \\ \hline & 0 \\ \hline & 0 \\ \hline \end{array}$ | $\left\|\begin{array}{l} 0 \\ \dddot{n} \\ a \end{array}\right\|$ | is | $\begin{array}{l\|l\|l} 8 \\ 8 \\ \hline \end{array}$ | $0$ | $0$ | $\begin{array}{l\|l\|l} 0 \\ \hline \\ \hline \end{array}$ | $\begin{array}{c\|c} 0 & 0 \\ 0 & 0 \end{array}$ | $0 \left\lvert\, \begin{aligned} & \circ \\ & 0 \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & \circ \\ & \vdots \\ & \vdots \end{aligned}\right.$ | $\left\|\begin{array}{c} 0 \\ \vdots \\ i \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \circ \\ & \stackrel{8}{6} \\ & \dot{a} \end{aligned}\right.$ | N |  | B |
| $\|\stackrel{\bullet}{6}\|$ | $\left\|\begin{array}{c} \stackrel{\circ}{\mathrm{N}} \\ \stackrel{y}{*} \end{array}\right\|$ | $\infty$ | $\left\|\begin{array}{c} 8 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{array}{l\|l\|l\|l\|l\|l\|} 0 \\ 0 & 0 \\ 0 \end{array}$ | $\begin{array}{l\|l} 0 & \infty \\ \infty & \underset{0}{\infty} \\ 0 \end{array}$ |  | $\begin{gathered} 0 \\ \hline \end{gathered}$ | $\begin{array}{l\|l\|l} 8 \\ 8 & 0 \\ \hline \end{array}$ |  | $\begin{array}{c\|c} \infty \\ \infty & \stackrel{\circ}{+} \\ \hline \end{array}$ | $\begin{array}{c\|c} \infty & \infty \\ \infty & \infty \\ i \end{array}$ | $\begin{array}{c\|c} \infty \\ \infty & \stackrel{0}{c} \\ \hline \end{array}$ |  | $\left\|\begin{array}{l} \infty \\ \stackrel{8}{i} \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ i \end{array}\right\|$ | $\cdots$ | $\stackrel{3}{0}$ | $2 \cdot \left\lvert\, \begin{aligned} & \infty \\ & 0 \\ & 0 \end{aligned}\right.$ | $0$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\|\begin{array}{c} 0 \\ i r \end{array}\right\|$ | $\frac{0}{\perp}$ | $\left\|\begin{array}{l} 0 \\ i \\ i \end{array}\right\|$ | $\stackrel{\circ}{-}$ | $\left\|\begin{array}{l} \tilde{0} \\ \end{array}\right\|$ |  |  |
| $\stackrel{\stackrel{\rightharpoonup}{0}}{0}$ | $\left\|\begin{array}{c} \infty \\ i \\ i \end{array}\right\|$ | $\stackrel{\circ}{\circ}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \vdots \\ & i \end{aligned}\right.$ | $0$ | $\underset{\sim}{c}$ | $\begin{array}{l\|l} 0 & \stackrel{8}{4} \\ \stackrel{n}{4} \end{array}$ | $\begin{aligned} & \circ \\ & \\ & \hline \end{aligned}$ | $\begin{array}{l\|l} 0 \\ 0 & \infty \\ 0 & \infty \end{array}$ |  | $\begin{array}{l\|l} 0 & 0 \\ 0 & \underset{\sim}{\infty} \\ \hline \end{array}$ | $\begin{array}{l\|l} +\infty & \infty \\ +\infty & \stackrel{\circ}{6} \end{array}$ | $\begin{array}{l\|l} \circ \\ 0 & \stackrel{0}{i} \\ \hline \end{array}$ | $\stackrel{\substack{\sim}}{\substack{0}}$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\stackrel{\circ}{\mathrm{i}}$ |  | $3 \mid \underset{\sim}{\mathrm{B}}$ | $0$ | $\underset{\text { in }}{\text { in }}$ | $0$ | $\begin{array}{l\|l\|l} 0 \\ 0 & \stackrel{\rightharpoonup}{2} \\ \hline \end{array}$ | $\frac{0}{2}$ | $0$ | $\left.\begin{array}{\|c} \circ \\ \hline 0 \end{array} \right\rvert\,$ | N |  |  |
| $\left\|\begin{array}{c} 0 \\ \hdashline \end{array}\right\|$ | $\left\|\begin{array}{l} \stackrel{\circ}{+} \\ + \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \infty \\ & \stackrel{+}{+} \\ & \hline \end{aligned}\right.$ | $\stackrel{\circ}{\circ}$ | $8$ | $\begin{array}{c\|c} \circ & \infty \\ \infty \\ \infty & 0 \\ \hline 0 \end{array}$ | $\begin{array}{l\|l} \infty & 8 \\ 0 & \stackrel{1}{4} \end{array}$ | $\begin{array}{l\|l} 0 \\ ⿻ ⿰ 丨 丨 八 口 \end{array}$ | $\begin{array}{l\|l\|l\|l\|l\|} \hline 0 \\ \hline \end{array}$ | $\begin{array}{l\|l\|l} 0 \\ 0 & 1 \\ 0 \end{array}$ |  | $\begin{array}{l\|l} \infty \\ \dot{*} & \stackrel{0}{i} \\ \hline \end{array}$ | $\begin{array}{l\|l} 0 & \stackrel{0}{i} \\ i \end{array}$ | $\stackrel{\circ}{\sim}$ | $\left\|\begin{array}{l} \infty \\ \stackrel{\infty}{i} \end{array}\right\|$ | $\left.\right\|_{\infty} ^{\bullet}$ | $8: \stackrel{N}{0}$ | $\begin{array}{l\|l} 3 \\ 0 & \stackrel{\circ}{\mathrm{~N}} \end{array}$ | $0$ | $\left\|\begin{array}{l} \infty \\ \hdashline \\ -\infty \end{array}\right\|$ | $\begin{array}{l\|l} \circ & \circ \\ \hline & \circ \\ \hline \end{array}$ |  | $\left\lvert\, \begin{aligned} & 0 \\ & \hdashline \\ & \infty \end{aligned}\right.$ | $\begin{array}{ll} 0 \\ \hline \end{array}$ | $\left\|\frac{6}{\infty}\right\|$ | N |  |  |
| $\left\|\begin{array}{c} 0 \\ \dot{3} \\ \dot{c} \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} \infty \\ \stackrel{\infty}{6} \\ 0 \\ 0 \end{array}\right\|$ | $\begin{gathered} 0 \\ -2 \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & \circ \\ & \hline \stackrel{\circ}{\circ} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{l\|l\|l} 8 \\ \hline & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{array}{c\|c} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 \end{array}$ |  | $0$ | $\begin{aligned} & \text { ol } \\ & \text { de } \end{aligned}$ | $\begin{gathered} 8 \\ \substack{0 \\ 0} \\ \hline 0 \\ \hline 0 \end{gathered}$ | $\begin{array}{l\|l} 0 & 0 \\ \hdashline & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{array}{c\|l} \circ & \infty \\ \circ & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\begin{array}{l\|l\|l} \infty & 0 \\ \infty & 0 \\ \infty & \dot{\alpha} \\ 0 & 0 \\ \hline \end{array}$ | $\begin{array}{l\|l} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\begin{array}{l\|l} 8 \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \circ \\ & + \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ |  |  |  | $0$ | $\begin{gathered} \circ \\ + \\ \vdots \\ 0 \\ 0 \end{gathered}$ | $\begin{gathered} 0 \\ \\ \\ \hline 0 \end{gathered}$ |  | $\stackrel{N}{\hat{\omega}}$ |  |  |
| $\left\|\begin{array}{c} \stackrel{\rightharpoonup}{n} \\ \stackrel{N}{0} \\ 0 \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \circ \\ \vdots \\ i_{2} \\ a_{2} \end{gathered}\right.$ | $\left\lvert\, \begin{aligned} & \circ \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{1}{0} \end{aligned}\right.$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $$ |  |  | $\begin{array}{l\|l} 0 \\ 0 \\ 0 & 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\underset{o}{\circ}$ | $\left.\begin{array}{c} 0 \\ \infty \\ 0 \end{array}\right)$ |  |  |  |  | $\begin{array}{l\|l} 8 \\ 0 & \stackrel{1}{2} \\ 0 & 2 \end{array}$ | $\stackrel{\circ}{\circ}$ | $\begin{array}{l\|l\|l} 0 & 0 \\ 0 & \mathrm{~N} \\ 0 & 0 \\ \hline 0 \end{array}$ |  | $\begin{array}{l\|l\|l} 0 & 0 \\ 0 & 0 \\ 0 \\ \hline 8.8 \end{array}$ | $\begin{array}{l\|l} 0 \\ 0 & 0 \\ 0 \\ 0 \\ 20 \\ 0 \end{array}$ |  | $\begin{array}{c\|c} 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ | $3$ | $\begin{array}{l\|l} 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ \hline 0 \end{array}$ | $\begin{array}{l\|l} 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\stackrel{\sim}{\sim}$ |  |  |
| $\left\|\begin{array}{c} e \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} \circ \\ i \\ i_{0} \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \infty \\ \infty \\ \infty \\ 0 \\ 0^{2} \end{gathered}\right.$ | $\begin{array}{l\|l\|} 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{c\|c\|c} 0 & 0 \\ \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  | $\begin{gathered} 0 \\ \vdots \\ \vdots \end{gathered}$ |  |  | $\begin{array}{l\|l} \circ \\ \dot{c} \\ \dot{c} \\ \hline \end{array}$ |  |  |  | $\begin{array}{l\|l\|l\|l\|l\|} \hline 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |  |  | $\begin{array}{l\|l} 8 \\ 0 & \stackrel{\rightharpoonup}{i} \\ 0 & 0 \\ 0 \end{array}$ | $\begin{array}{\|l\|l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{aligned} & 0 \\ & \vdots \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  | $\left\lvert\, \begin{gathered} N \\ \underset{U}{0} \\ \hline \end{gathered}\right.$ |  |  |
| $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{gathered} \circ \\ \stackrel{3}{4} \\ 0 . \\ 0 \end{gathered}$ | $\begin{array}{\|c} 0 \\ y_{1} \\ c_{0} \\ \hline \end{array}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{l\|l\|l} 0 \\ 0 & 0 \\ 0 & 20 \\ 0 & 0 \end{array}$ |  |  | $\begin{array}{l\|l} 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & + \\ \hline 0 \\ \hline \end{array}$ | $\begin{array}{l\|l\|l\|l} 0 \\ 0 & 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ |  |  |  | $\begin{aligned} & \text { for } \\ & \stackrel{c}{c} \\ & \hline \end{aligned}$ | $\begin{array}{llll} 0 & 0 \\ & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ |  | $\begin{aligned} & 3 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{aligned} & 3 \\ & 0 \\ & 0 \\ & 6 \\ & 30 \\ & 90 \end{aligned}$ | $\mathfrak{l}$ | $\begin{gathered} 8 \\ \circ \\ 0 \\ 0 \end{gathered}$ |  | $\begin{gathered} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{array}{l\|l} 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ |  | $$ |  |  |
| $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} \circ \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}\right.$ | $\left\lvert\, \begin{gathered} 8 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}\right.$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \hline 0 \\ & \hline 0 \end{aligned}$ |  | $\begin{array}{ll} 8 \\ 0 & 0 \\ 0 & \stackrel{0}{0} \\ 0 & 0 \end{array}$ | $\begin{array}{l\|l} 8 \\ 0 & 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{array}{l\|l} 0 \\ 0 & 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{array}{ll} 0 & 0 \\ 0 \\ 0 & 0 \\ 0 \end{array}$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ |  |  | $\begin{array}{c\|c} \circ \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\begin{aligned} & R \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}\right.$ |  |  |  | $\begin{aligned} & 3 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{ccc} 0 \\ 0 & 0 \\ 0 \\ 0 & 0 \\ 0 \\ 0 \end{array}$ |  |  | $\begin{array}{l\|l} 0 \\ 0 \\ 0 & 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{array}{ll} 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ | $\stackrel{\sim}{\square}$ |  |  |

TABLE III
Illustrated Ratio Study Statistics

|  | (1.) |  | (2.) | (3.) | (4.) | (5.) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Property |  | Sale | Assessed | Ratio | Absolute |  |
|  | Number |  | Price | Value | A/S \% | $\begin{aligned} & \text { Deviation } \\ & \text { from } \end{aligned}$ |  |
|  |  |  |  |  |  | Median |  |
|  | 1 |  | 28,000 | 22,400 | 80\% | 20\% |  |
|  | 2 |  | 22,000 | 19,250 | 88\% | 12\% |  |
|  | 3 |  | 63,500 | 55,575 | 88\% | 12\% |  |
|  | 4 |  | 55,900 | 51,700 | 92\% | 7\% |  |
|  | 5 |  | 20,000 | 19,000 | 95\% | 5\% |  |
|  | 6 |  | 21,000 | 20,475 | 98\% | 2\% |  |
|  | 7 |  | 80,000 | 80,000 | 100\% | 0\% |  |
|  | 8 |  | 40,000 | 40,000 | 100\% | 0\% |  |
|  | 9 |  | 33,000 | 33,300 | 101\% | 1\% |  |
|  | 10 |  | 45,000 | 46,125 | 103\% | 3\% |  |
|  | 11 |  | 24,000 | 25,200 | 105\% | 5\% |  |
|  | 12 |  | 39,000 | 41,925 | 108\% | 8\% |  |
|  | 13 |  | 37,000 | 41,625 | 113\% | 13\% |  |
|  | 14 |  | 40,300 | 45,800 | 114\% | 14\% |  |
|  | 15 |  | 51,000 | 59,925 | 118\% | 18\% |  |
|  | TOTAL |  | 599,700 | 602,300 | 1500\% | 120\% |  |
| Average Ratio |  | $=$ | Total of Ratios (4.) | $\div$ | Number of Sales (1.) |  |  |
|  |  |  | 1500\% | $\div$ | 15 | = | 100\% |
| Weighted Ratio |  | $=$ | Total of Assessed Values (3.) | $\div$ | Total of Sale Prices (2.) |  |  |
|  |  |  | $602,300$ | $\div$ | $599,700$ | $=$ | 100\% |
| Average Deviation |  | $=$ | Total Deviations (5.) | $\div$ | Number of Sales (1.) |  |  |
|  |  |  | 120\% | $\div$ | 15 | = | 8\% |
| Median Ratio |  | = | Middle Value of Data Array |  |  | = | 100\% |
|  |  |  | (i.c. $100 \%$ |  |  |  |  |
|  |  |  | (i.e. property \#8) |  |  |  |  |
| Coefficient of |  | $=$ | Average Deviation (5.) | $\div$ | Median Ratio (4.) |  |  |
| Dispersion |  |  | 8\% | $\div$ | 100\% | $=$ | 7.98 |
| Price Related |  | $=$ | Average Ratio (4.) | $\div$ | Weighted Ratio |  |  |
| Differential |  |  | 100\% | $\div$ | 100\% | $=$ | 1.00 |


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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 006＇tャて\＄ | £9＊0I | 0I＊0 | Z0＊ | $10 \cdot L$ | \％9＊9 | \％どて6 | \％I＇ャ6 |  | \＆てZ |  |
| ¢ZL＇E9I\＄ | $9 \varepsilon^{*} \varepsilon 1$ | ZI＇0 | I0＊ | E＊6 | \％88 | \％0＊I6 | \％どを6 | \％S＇I6 | て£ย | 0э！Шоэ！ |
| 000＇¢9I\＄ | しがII | 01＊0 | 00＊ | ¢E．8 | \％L＇L | \％668 | \％I＇Z6 | \％ナ＊06 | I8t |  |
| 000＇6It\＄ | $\dagger S^{\prime} \dagger \mathrm{I}$ | †I 0 | I0＊ | 8800 | \％S＇01 | \％ガく6 | \％696 | \％ガ96 | 96I | ${ }^{10 q / 81}$ |
| $000{ }^{\circ} \varepsilon$ I ${ }^{\text {d }}$ | Z6＊ | 90.0 | $00^{\circ} \mathrm{I}$ | 91＇\％ | \％0＇t | \％でと6 | \％6 ${ }^{\circ} 6$ | \％どを6 | $0 \varepsilon$ | р๐s．ıш0S |
| 000＇ऽऽE\＄ | $\varepsilon L^{\circ} \mathrm{L}$ | L0．0 | 00． 1 | $9 L^{\circ} \mathrm{S}$ | $\%{ }^{\circ} \mathrm{C}$ | \％8「て6 | \％0＾ャ6 | \％でと6 | ¢†¢ |  |
| 0¢でてもて\＄ | I $8^{\circ} \mathrm{L}$ | 80.0 | I0＊ | てI＇t | \％0＊$\dagger$ | \％9｀¢6 | \％096 | \％ど96 | て\＆I | s，дuиу uәәпठ |
| 000＇もをと\＄ | ［［ ${ }^{\circ} 0$ I | $60^{\circ} 0$ | $00^{\circ} \mathrm{I}$ | It＊ | \％69 | \％${ }^{\text {c }}$ ¢6 | \％ャ・¢6 | \％9｀¢6 | ャ9L＇E |  |
| 000＇SLt\＄ | 69001 | 0100 | $00^{\circ} \mathrm{I}$ | L6．9 | \％ガ9 | \％6 16 | \％ガて6 | \％0＾て6 | $06 \varepsilon^{\text {¢ }}$ ¢ |  |
| 00S＇89て\＄ | LE＇II | I ${ }^{\circ} 0$ | 20＇I | 28＊9 | \％9＊9 | \％8 $\downarrow 6$ | \％ガL6 | \％ど96 | て8 | ұиәу |
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| 000＇sç\＄ | 6¢ ${ }^{\circ}$ 0I | 01＊0 | 00＊ | $\varepsilon L^{\circ} L$ | \％でし | \％8＇I6 | \％6「て6 | \％0＾て6 | 06L＇I | צэ！．әрәлы |
| 006＇69 I\＄ | L6＇${ }^{\circ}$ I | $\varepsilon L^{\circ} 0$ | 20＇I | \＆ऽ＇6 | \％1「6 | \％6 26 | \％9｀¢6 | \％8＇ャ6 | LSI |  |
| $000 \times$ ¢E§ | て¢＇L | L0．0 | $00^{\circ} \mathrm{I}$ | LS＇S | \％で「 |  | \％8＇£6 | \％でを6 | 6SS | SวIRYつ |
| 000＇ऽEz\＄ | ¢で0I | $60^{\circ} 0$ | 00＊ | I $8^{\circ} \mathrm{L}$ | \％ガL | \％¢＇06 | \％でャ6 | \％9＊06 | ¢87 | I！つ入 |
| 000＇s0を\＄ | 60\％01 | $60^{\circ} 0$ | I0＊ | E9 ${ }^{\circ}$ | \％I＇L | \％どを6 | \％¢｀を6 | \％0＊$\dagger 6$ | 909 | II0．IIP？ |
| 000＇6IZ\＄ | 00．01 | 01．0 | 00＊ | Lt＊ | \％${ }^{\circ} \mathrm{L}$ L | \％9＊96 | \％と｀¢6 | \％ど96 | ¢0I | әu！！0．IEว |
| 00S＇69E\＄ | $60^{\circ} \mathrm{L}$ | L0．0 | 00＊ | $6 t^{\circ} \mathrm{S}$ | \％${ }^{\text {＇S }}$ | \％ $\mathrm{c}^{\bullet}$ ¢6 | \％0｀¢6 | \％¢｀¢6 | LセE |  |
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| $000 \times$ ¢ 2 \＄ | 8で01 | 010 | 00＊ | $69^{\circ} \mathrm{L}$ | \％でし | \％9＊と6 | \％6＇${ }^{\circ} 6$ | \％6＇${ }^{\text {\％}}$ | £ Z6＇${ }^{\circ}$ | Iəpun．sV วuuv |
| 000＇EZIS | $69^{\circ}+$ | ＋0．0 | 00＊ | LE＇ $\mathcal{L}$ | $\%$ \％＇を | \％ど¢6 | \％8＇¢6 | \％で¢6 | $\dagger$ ¢ | Kuboi？ |
|  |  | ио！̣⿺！ләТ <br> p．rяривłS |  |  |  |  | $\begin{gathered} \text { о!̣еप } \\ \text { u飞!pəW } \end{gathered}$ |  |  |  |

TABLE IV-B
Statewide Residential Ratio Study Frequency Statistics
Average Ratio

| Average Ratio |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Total of Ratios | $=$ | $23608.48$ | = | 92.7\% |
| Weighted Ratio |  |  |  |  |
| Total Assessed Values Total Sales Prices | = | $\frac{8,824,772,100}{9,555,520,290}$ | $=$ | 92.4\% |
| Average Deviation |  |  |  |  |
| Total Deviations Number of Sales | = | $\frac{2,034}{25,456}$ | = | 8.0\% |

Number of Sales

Coefficient of Dispersion
Average Absolute Deviation = Median Ratio
$8.0 \%$
$93 \%$
8.55

Price Related Differential

| Average Ratio |  |  |  |
| :--- | :--- | :--- | :--- |
| Weighted Ratio | $=$ | $92.74 \%$ | $92.35 \%$ |$\quad 1.00$

## Table V

## Commercial Ratio Study 2017

The table below shows statistics on arms-length sales between July 1, 2016 and June 30, 2017 of commercial property in assessment Group 2. Ratios compare the Department's January 1, 2017, value to the actual sale price.

Ratio statistics are shown for all jurisdictions, even where the number of sales is so small that there is not a sufficient sample to provide accurate statistics. In cases where there are fewer than 10 sales, the ratio statistics are not used to calculate the base (Table I).

|  | Number <br> of Sales | Total Assessed <br> Values | Total Sales <br> Prices | Weighted <br> Ratio | Average <br> Ratio | Median <br> Ratio |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Allegany | 8 | $7,686,600$ | $7,864,200$ | $97.7 \%$ | $96.7 \%$ | $96.9 \%$ |
| Anne Arundel | 66 | $361,654,700$ | $395,641,381$ | $91.4 \%$ | $93.0 \%$ | $95.8 \%$ |
| Baltimore City | 142 | $290,863,300$ | $347,874,908$ | $83.6 \%$ | $85.1 \%$ | $91.2 \%$ |
| Baltimore County | 52 | $80,918,500$ | $87,960,276$ | $92.0 \%$ | $99.3 \%$ | $98.9 \%$ |
| Calvert | 2 | 798,000 | $1,015,000$ | $78.6 \%$ | $71.7 \%$ | $71.7 \%$ |
| Caroline | 5 | $6,055,000$ | $6,353,000$ | $95.3 \%$ | $100.5 \%$ | $98.6 \%$ |
| Carroll | 5 | $1,874,400$ | $1,878,000$ | $99.8 \%$ | $99.5 \%$ | $100.1 \%$ |
| Cecil | 10 | $10,392,600$ | $12,155,921$ | $85.5 \%$ | $92.4 \%$ | $94.0 \%$ |
| Charles | 7 | $3,457,700$ | $3,899,180$ | $88.7 \%$ | $92.3 \%$ | $95.7 \%$ |
| Dorchester | 7 | $1,736,300$ | $2,001,987$ | $86.7 \%$ | $91.4 \%$ | $96.2 \%$ |
| Frederick | 44 | $22,099,000$ | $24,284,999$ | $91.0 \%$ | $92.6 \%$ | $94.2 \%$ |
| Garrett | 2 | 314,100 | 315,000 | $99.7 \%$ | $104.3 \%$ | $104.3 \%$ |
| Harford | 12 | $7,608,500$ | $8,289,000$ | $91.8 \%$ | $95.1 \%$ | $96.9 \%$ |
| Howard | 20 | $85,622,200$ | $83,290,312$ | $102.8 \%$ | $99.6 \%$ | $96.5 \%$ |
| Kent | 1 | 196,700 | 219,000 | $89.8 \%$ | $89.8 \%$ | $89.8 \%$ |
| Montgomery | 42 | $203,937,500$ | $256,707,169$ | $79.4 \%$ | $92.5 \%$ | $95.5 \%$ |
| Prince George's | 52 | $218,274,100$ | $250,420,755$ | $87.2 \%$ | $90.0 \%$ | $90.6 \%$ |
| Queen Anne's | 3 | 946,400 | 985,531 | $96.0 \%$ | $93.4 \%$ | $98.6 \%$ |
| St. Mary's | 5 | $2,456,200$ | $2,296,800$ | $106.9 \%$ | $107.5 \%$ | $106.3 \%$ |
| Somerset | 2 | 534,300 | 506,500 | $105.5 \%$ | $102.8 \%$ | $102.8 \%$ |
| Talbot | 2 | $1,354,900$ | $1,485,000$ | $91.2 \%$ | $85.6 \%$ | $86.1 \%$ |
| Washington | 50 | $34,235,900$ | $38,324,147$ | $89.3 \%$ | $91.7 \%$ | $92.7 \%$ |
| Wicomico | $6,278,500$ | $8,130,850$ | $77.2 \%$ | $88.3 \%$ | $93.6 \%$ |  |
| Worcester | 20 | 18 | $19,958,800$ | $23,763,900$ | $84.0 \%$ | $92.3 \%$ |
| Statewide | $\mathbf{5 8 0}$ | $\mathbf{\$ 1 , 3 6 9 , 2 5 4 , 2 0 0}$ | $\mathbf{\$ 1 , 5 6 5 , 6 6 2 , 8 1 6}$ | $\mathbf{8 7 . 5 \%}$ | $\mathbf{9 1 . 5 \%}$ | $\mathbf{9 5 . 3 \%}$ |




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